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Performance characteristics of an irreversible thermally driven Brownian microscopic heat engine

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Abstract. Brownian particles moving in a spatially asymmetric but periodic potential (ratchet), with an external load force and connected to an alternating hot and cold reservoir, are modeled as a microscopic heat engine, referred to as the Brownian heat engine. The heat flow via both the potential energy and the kinetic energy of the particles are considered simultaneously. The forward and backward particle currents are determined using an Arrhenius' factor. Expressions for the power output and efficiency are derived analytically. The maximum power output and efficiency are calculated. It is expounded that the Brownian heat engine is always irreversible and its efficiency cannot approach the efficiency η_C of the Carnot heat engine even in quasistatic limit. The influence of the main parameters such as the load, the barrier height of the potential, the asymmetry of the potential and the temperature ratio of the heat reservoirs on the performance of the Brownian heat engine is discussed in detail. It is found that the Brownian heat engines may be controlled to operate in different regions through variation of some parameters.

PACS. 05.40.Jc Brownian motion – 05.70.-a Thermodynamics – 05.60.-k Transport processes

1 Introduction

Recently, Brownian (microscopic) heat engines have attracted much attention for the utilization of energy resources available at the microscopic scale and the miniaturization of devices demanding tiny engines that operate at the same scale. A Brownian heat engine (motor), which appeared in Feynman's famous textbook for the first time as a thermal ratchet [1], is a machine that can rectify thermal fluctuation to produce a directed current and has been studied and applied in some research fields [2–5]. Usually, Brownian heat engines are spatially asymmetric but periodic structures, in which the transportation of Brownian particles is induced by some non-equilibrium processes [6,7]. Typical examples are external modulation of an underlying potential [8–11], an activation of an external force [12–14], a non-equilibrium chemical reaction coupled to a change of the potential [15] or a contact with the reservoirs at different temperatures [16–18].

Brownian heat engines driven by a contact with the reservoirs at different temperatures, i.e., thermally driven Brownian heat engines, were first proposed by Buttiker [16], van Kampen [17], and Landauer [18] and have been investigated since by many researchers [19–25]. Derenyi and Astumian [19] found that the efficiency of thermally driven Brownian heat engines can approach the efficiency of the Carnot heat engine under the quasistatic limit. The same results were also obtained in Matsuo and Sasa's work [21] using stochastic energetic theory.

Most of the works mentioned above are limited to the case of heat flow due to the change of the potential energy of Brownian particles, while the heat flow due to the change of the kinetic energy of Brownian particles has not been taken into account. It is very necessary and significant to consider simultaneously the heat flows via both the potential and kinetic energies in the Brownian heat engines, because the heat flow via the kinetic energy is irreversible and results in a large influence on the efficiency of the Brownian heat engines.

In the present paper, we will consider the heat flows via the potential and kinetic energies of Brownian particles and derive the expressions of the power output and efficiency of an irreversible Brownian heat engine. Moreover, the influence of some main parameters on the performance of the Brownian heat engine is analyzed, and consequently, some novel results, which can reveal the general performance characteristics of the Brownian heat engine, are obtained. Finally, it is proposed that the Brownian heat engine may be controlled to operate in special chosen states through the adjustment of some parameters.

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2 A thermally driven Brownian heat engine

Consider Brownian particles moving in a spatially asymmetric but periodic potential (ratchet) with an external load force f. The particles are periodically in contact with two heat reservoirs along the space coordinate [19,23,24], as shown in Figure 1, where $\dot{N}_+$ and $\dot{N}_-$ are, respectively, the numbers of forward and backward jumps per unit time, T_H and T_C are, respectively, the temperatures of the hot and cold reservoirs, L_1 and L_2 are, respectively, the lengths of the left and right parts of the ratchet, $L = L_1 + L_2$ is the period length of the potential, and E is the barrier height of the potential. It should be pointed out that the continuous piecewise linear potential shown in Figure 1 is only characterized by the parameters E, L_1 and L_2 and is just one of the feasible selections, which may be taken as an example for readers' convenience. There are two different driving factors for Brownian particles. The first one is noise-induced transport, namely, the ratchet effect. The second one is the temperature difference that makes the particles move from the high- to the low-temperature reservoir. $E + fL_1$ is the energy needed by a particle for a forward jump, while $E - fL_2$ is the energy needed by a particle for a backward jump. The left part (Region I or I′) of a period ratchet is contacted with the hot reservoir at temperature T_H and the right one (region II or II′) is contacted with the cold reservoir at temperature T_C. It is assumed that the rates of both forward and backward jumps are proportional to the corresponding Arrhenius' factor [22] and the system is in a state of stable flow, so that the numbers of forward and backward jumps per unit time are, respectively, determined by

$$\dot{N}_+ = (1/t)\exp[-(E + fL_1)/(k_B T_H)] \tag{1}$$

and

$$\dot{N}_- = (1/t)\exp[-(E - fL_2)/(k_B T_H)], \tag{2}$$

where k_B is the Boltzmann constant and t is a proportionality constant with a time dimension.

If $\dot{N}_+ > \dot{N}_-$, the ratchet works as a two-reservoir heat engine. When Brownian particles move in different regions, the change of the potential energy will result in heat exchange between the heat engine and the heat reservoirs. The heat flow from the hot reservoir to the heat engine via potential is

$$\dot{Q}_H^{pot} = (\dot{N}_+ - \dot{N}_-)(E + fL_1) \tag{3}$$

and the heat flow from the heat engine to the cold reservoir via potential is

$$\dot{Q}_C^{pot} = (\dot{N}_+ - \dot{N}_-)(E - fL_2). \tag{4}$$

The heat flow resulting from the change of the kinetic energy of Brownian particles is much more complicated [19]. When the particle lies in a region, it is in equilibrium with a heat reservoir. According to the theory of energy equipartition, the average kinetic energy per particle is equal to $1/2k_BT$. When the forward particles leave region I (the hot reservoir) and enter region II (the cold reservoir), each particle will release $1/2k_B(T_H - T_C)$ energy to the cold reservoir to reduce its average kinetic energy. Thus, owing to the change of the kinetic energy of Brownian particles, the first part of heat flow from the heat engine to the cold reservoir is $\dot{Q}_{C1}^{kin} = 1/2\dot{N}_+k_B(T_H - T_C)$. Similarly, when the backward particles leave region II (the cold reservoir) and enter region I (the hot reservoir), each particle will absorb $1/2k_B(T_H - T_C)$ energy from the hot reservoir to raise its average kinetic energy. The first part of heat flow from the hot reservoir to the heat engine is $\dot{Q}_{H1}^{kin} = 1/2\dot{N}_-k_B(T_H - T_C)$. In order to keep the heat engine operate continuously and stably, the particles in region I must be supplied from the neighbor region II′, so each particle will pick up $1/2k_B(T_H - T_C)$ energy from the hot reservoir to raise its average kinetic energy. Obviously, owing to the change of the kinetic energy of Brownian particles, the second part of heat flow from the hot reservoir to the heat engine is $\dot{Q}_{H2}^{kin} = 1/2\dot{N}_+k_B(T_H - T_C)$. Similarly, the particles in region II must be supplied from neighbor segment I′ and each particle will release $1/2k_B(T_H - T_C)$ energy to the cold reservoir to reduce its average kinetic energy. Consequently, the second part of heat flow from the heat engine to the cold reservoir is $\dot{Q}_{C2}^{kin} = 1/2\dot{N}_-k_B(T_H - T_C)$. It is seen from the above analysis that the total heat flow from the hot reservoir to the heat engine due to the change of the kinetic energy of Brownian particles is equal to that from the heat engine to the cold reservoir, i.e.,

$$\begin{aligned}\dot{Q}_H^{kin} &= \dot{Q}_{H1}^{kin} + \dot{Q}_{H2}^{kin} = \frac{1}{2}k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)\\ &= \dot{Q}_{C1}^{kin} + \dot{Q}_{C2}^{kin} = \dot{Q}_C^{kin} \equiv \dot{Q}^{kin}.\end{aligned} \tag{5}$$

It can be seen from equation (5) that the energy $1/2k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)$ is transferred completely from the heat reservoir to the cold reservoir. This indicates the inherently irreversible nature of this heat flow.

It is found from the above results that the total heat flow absorbed from the hot reservoir is

$$\begin{aligned}\dot{Q}_H = \dot{Q}_H^{pot} + \dot{Q}_H^{kin} &= (\dot{N}_+ - \dot{N}_-)(E + fL_1)\\ &+ \frac{1}{2}k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)\end{aligned} \tag{6}$$

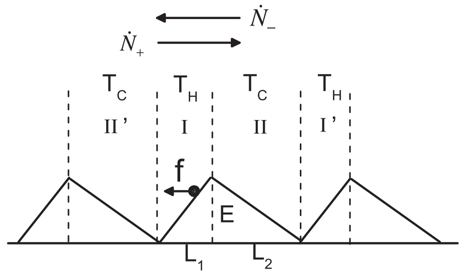


Fig. 1. Schematic diagram of a thermally driven Brownian heat engine.

$$\eta = \frac{\dot{W}}{\dot{Q}_H} = \frac{(e^{-(\varepsilon+x\mu)} - e^{-(\varepsilon-x+x\mu)/\tau})x}{(e^{-(\varepsilon+x\mu)} - e^{-(\varepsilon-x+x\mu)/\tau})(\varepsilon + x\mu) + \frac{1}{2}(e^{-(\varepsilon+x\mu)} + e^{-(\varepsilon-x+x\mu)/\tau})(1-\tau)}, \tag{9}$$

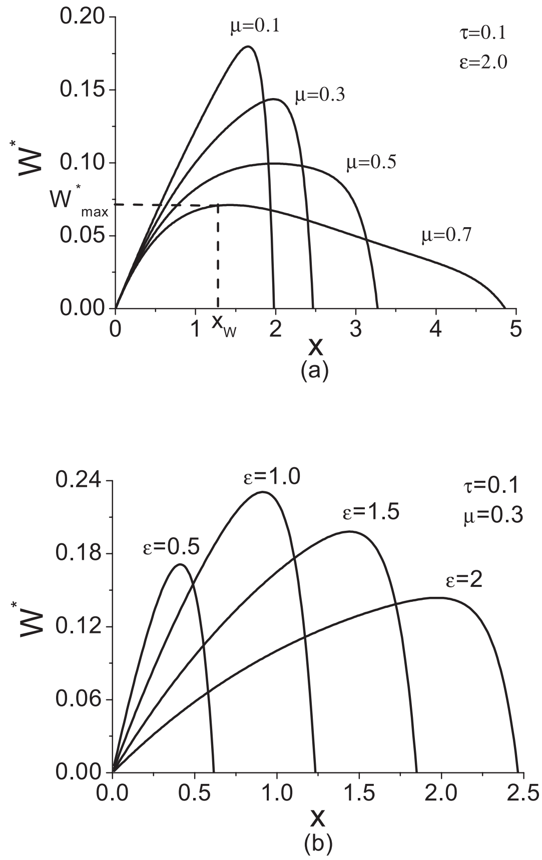


Fig. 2. Dimensionless power output W^* varying with the dimensionless load x for some given parameters: (a) $\varepsilon = 2.0$ and $\tau = 0.1$, (b) $\mu = 0.3$ and $\tau = 0.1$.

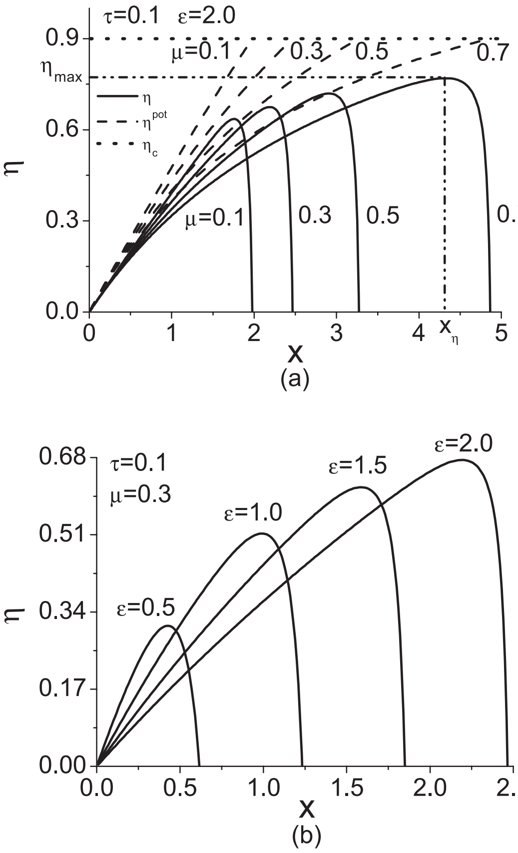


Fig. 3. Efficiency η varying with the dimensionless load x for some given parameters: (a) $\varepsilon = 2.0$ and $\tau = 0.1$, (b) $\mu = 0.3$ and $\tau = 0.1$. The dotted line indicates the efficiency η_C of the Carnot heat engine and the dashed lines indicate the efficiency η^{pot} of the Brownian heat engine, which is defined by $\eta^{pot} = \dot{W}/\dot{Q}_H^{pot}$ [19].

and that the total heat flow released to the cold reservoir is

$$\dot{Q}_C = \dot{Q}_C^{pot} + \dot{Q}_C^{kin} = (\dot{N}_+ - \dot{N}_-)(E - fL_2) + \frac{1}{2}k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C). \tag{7}$$

Thus, the power output and efficiency of the Brownian heat engine may be, respectively, expressed as

$$\dot{W} = \dot{Q}_H - \dot{Q}_C = (\dot{N}_+ - \dot{N}_-)fL = k_B T_H (e^{-(\varepsilon+x\mu)} - e^{-(\varepsilon-x+x\mu)/\tau})x/t \tag{8}$$

and

see equation (9) above

where $x = fL/(k_B T_H)$ is the dimensionless load, $\varepsilon = E/(k_B T_H)$ indicates the dimensionless barrier height of the potential, $\tau = T_C/T_H$ is the temperature ratio of the cold to the hot reservoir and $\mu = L_1/L$. Equations (8) and (9) show clearly that the heat flow due to the change of the kinetic energy of Brownian particles does not affect the power output of the Brownian heat engine, but it affects the efficiency of the Brownian heat engine.

3 The maximum power output and efficiency

It is clearly seen from equations (8) and (9) that when $x = 0$ and $x = \frac{(1-\tau)\varepsilon}{(\tau-1)\mu+1} \equiv x_{\max}$, both the power output and efficiency are equal to zero. Their physical meanings are very clear. When $x = 0$, the engine runs without a load, so that $\dot{Q}_H$ is equal to $\dot{Q}_C$, which indicates that the heat absorbed from the hot reservoir by the heat engine is completely passed to the cold reservoir and neither power output nor efficiency is obtained. When $x = x_{\max}$, the net current of the particles ($\dot{N} = \dot{N}_+ - \dot{N}_-$) is equal to zero, so that $\dot{Q}_H$ is equal to $\dot{Q}_C$ once again and neither power output nor efficiency is obtained. Thus, if and only if the relation $0 < x < x_{\max}$is satisfied, can the ratchet work as a two-reservoir heat engine.

Using equations (8) and (9), one can plot the $W^* \sim x$, $\eta \sim x$ curves of the Brownian heat engine, as shown in Figures 2 and 3, respectively, where $W^* = \dot{W}t/(k_B T_H)$. The curves in Figure 2 show that the power output first

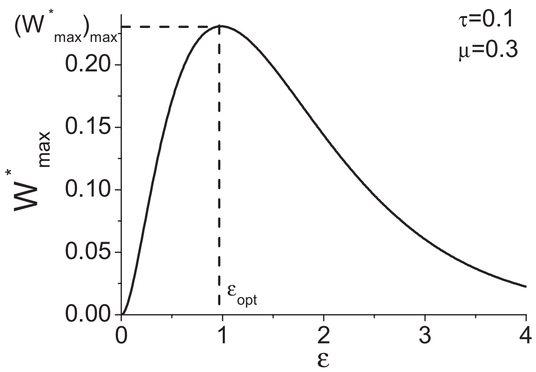


Fig. 4. The $W^*_{\max} \sim \varepsilon$ curve for some given parameters $\mu = 0.3$ and $\tau = 0.1$.

increases and then decreases as x increases. When $x = x_W$, the power output attains its maximum. It is easily found from equation (8) and its extremal condition that x_W is determined by

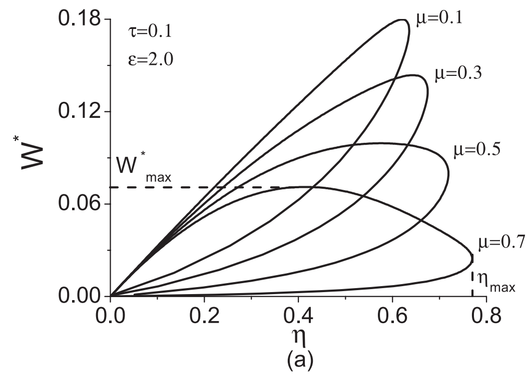
$$e^{(\varepsilon+x\mu)-(\varepsilon-x+x\mu)/\tau} = (\tau - \tau x\mu)/(\tau + x - x\mu). \quad (10)$$

The curves in Figure 2a show that $W^*_{\max}$ will decrease and $x_{\max}$ will increase when μ increases, while x_W is not a monotonic function of μ. The curves in Figure 2b show that that $x_{\max}$ and x_W will increase when the dimensionless barrier height of the potential ε increases, while the dimensionless maximum power output $W^*_{\max}$ is not a monotonic function of ε, as shown in Figure 4. It is seen from Figure 4 that when $\varepsilon \to 0$ and $\varepsilon \to \infty$, $W^*_{\max} = 0$. When $\varepsilon = \varepsilon_{opt}$, $W^*_{\max} = (W^*_{\max})_{\max}$. In fact, when $\varepsilon \to 0$, the ratchet effect disappears and no particle currents occur. When $\varepsilon \to \infty$, the particles cannot pass the barrier of the potential and no particle currents occur.

The curves in Figure 3 show that the efficiency first increases and then decreases as x increases. When $x = x_\eta$, the efficiency of the Brownian heat engine attains its maximum $\eta_{\max}$, where $\eta_{\max}$ is only the maximum efficiency for a set of particularly chosen parameters μ, τ and ε, but not the general maximum efficiency of this system. Further analysis shows that $\eta_{\max}$will increase with the increase of μ and ε for a given τ. It is easily found from equation (9) and its extremal condition that x_η is determined by

$$Ae^{-2(\varepsilon+x\mu)} + Be^{-2(\varepsilon-x+x\mu)/\tau} + Ce^{-(\varepsilon+x\mu)-(\varepsilon-x+x\mu)/\tau} = 0, \quad (11)$$

where $A = \varepsilon + (1-\tau)/2$, $B = \varepsilon - (1-\tau)/2$ and $C = -2\varepsilon - x\mu + x\mu\tau - (1-\mu)(1-\tau)x/\tau$. The curves in Figure 3 also show that when the heat flow via kinetic energy is considered, the efficiency η cannot approach the efficiency η_C of the Carnot heat engine, because there exists an irreversible heat loss due to the change of kinetic energy. When the heat flow via kinetic energy is not considered [19], the efficiency η^{pot} of the Brownian heat engine is a monotonically increasing function of x. When $x = x_{\max}$, i.e., the net current of the particles is equal to zero, η^{pot} approaches η_C [21], as shown by the dashed curves in Figure 3a. In addition, it is seen from Figure 3 that the efficiency is an increasing function of μ and ε.



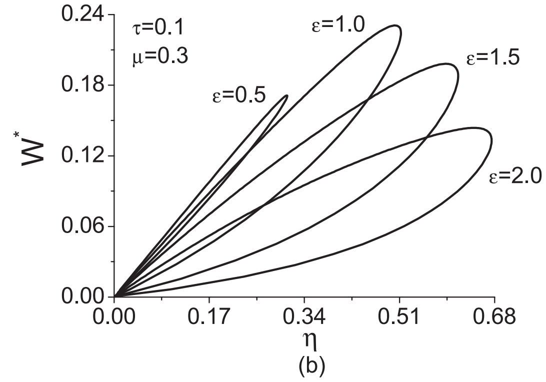


Fig. 5. Dimensionless power output W^* varying with the efficiency η for some given parameters: (a) $\varepsilon = 2.0$ and $\tau = 0.1$, (b) $\mu = 0.3$ and $\tau = 0.1$.

4 The power output versus efficiency characteristics

In order to understand further the general characteristics of the Brownian heat engine, equations (8) and (9) can be used to generate the power output versus efficiency curves, as shown in Figure 5. It is seen from Figures 2, 3 and 5 that when the heat engine is operated in the region of $x \le x_W$ or $x \ge x_\eta$, the power output will decrease as the efficiency decreases. When the heat engine is operated in the region of $x_W \le x \le x_\eta$, the power output will increase as the efficiency decreases, and vice versa. Through the choice of the parameters x, μ, ε and τ, the Brownian heat engines may be controlled to operate under different behaviour regimes.

5 Conclusions

The performance of a one dimensional thermally driven Brownian heat engine is studied. It is found that the heat flow $1/2k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)$ due to the change of the kinetic energy of the particles is transferred completely from the hot to the cold reservoir, the Brownian heat engine is always irreversible and its efficiency cannot approach the efficiency η_C of the Carnot heat engine even in the quasistatic limit. It is also found that the influence of the load, the barrier height of the potential, the asymmetry of the potential and the temperature ratio of the heat reservoirs on the performance of the Brownian heat engine is obvious. Brownian heat engines may be controlled to operate under different regimes through the choice of the parameters x, μ, ε and τ. When $x = x_W$ and $\varepsilon = \varepsilon_{opt}$ are chosen, the Brownian heat engines are operated in the state of maximum power output. When $x = x_\eta$ is chosen, the Brownian heat engines are operated in the state of maximum efficiency.

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